

The Supersymmetric Singlet Majoron Model and the General Upper Bound on the Lightest Higgs Boson Mass

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Abstract

An upper bound on the tree-level mass of the lightest Higgs boson of the Supersymmetric Singlet Majoron Model is obtained. Contrary to some recent claims, it is shown to be of the same form as the general mass bound previously calculated for supersymmetric models with an extended Higgs sector. Soft-breaking masses or exotic vacuum expectation values do not enter in the tree-level bound [which is only controlled by the electroweak scale (M_Z)] and also decouple from the most important radiative corrections to the bound (the ones coming from the top-stop sector). The derivation of the upper bound for general Supersymmetric Models is reviewed in order to clarify its range of applicability.

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The search for a Higgs boson is one of the most challenging goals for existing and planned accelerators. The discovery of such a fundamental scalar would be the first step in understanding the elusive mechanism of electroweak symmetry breaking. In the framework of supersymmetric theories the Higgs sector is particularly constrained and provides a unique ground for checking whole classes of supersymmetric models. In contrast with the arbitrariness in the masses of most of the new particles predicted by supersymmetry (which are only weakly restricted by naturalness criteria) it seems to be a general feature of Supersymmetric Standard Models the presence of a light Higgs particle in the spectrum (with mass of order M_Z even in the limit of unnaturally large supersymmetric masses).

As is well known, in the Minimal Supersymmetric Standard Model the tree level mass m_h of the lightest Higgs boson is bounded by $M_Z |\cos 2\beta|$. Radiative corrections to the mass of this Higgs boson can be large if the mass of top and stops is large, and the tree level bound can be spoiled [1]. After including next-to-leading log corrections [2] the numerical bound $m_h < 140 \text{ GeV}$ (for a top mass below 190 GeV and stops not heavier than 1 TeV) is found, so that, even if the lightest Higgs can escape detection at LEP-200 its mass is always of the order of the electroweak scale (and the dependence on the soft breaking scale is only logarithmic). Similar bounds have been calculated for extended supersymmetric models. An analytical upper bound on the tree-level mass of the lightest Higgs boson (LHB) is known for very general Supersymmetric Standard Models with extended Higgs or gauge sectors [3, 4]. This bound depends on the electroweak scale (given by M_Z) and on the new Yukawa or gauge couplings that appear in the theory. Numerical bounds can be obtained by placing limits on these unknown parameters (*e.g.* assuming that the theory remains perturbative up to some high scale). These bounds are typically greater than the MSSM bound but still of order M_Z .

In a recent paper [5] a bound on the lightest Higgs boson mass was calculated in a particular supersymmetric extended model with spontaneous R-parity breaking, the supersymmetrized Singlet Majoron Model (SSMM). The obtained bound was found to be qualitatively different from the general bounds of [3, 4]. In contrast with these general bounds it was found a dependence on exotic vacuum expectation values (VEVs) that are naturally of the order of the SUSY breaking scale so that the bound is no longer controlled by M_Z , although it turned out to be numerically very similar to the MSSM bound due to the smallness of some Yukawa couplings. Theoretically this is a disturbing result, because it leaves open the possibility of finding similar models in which the mass of the LHB is much larger than M_Z (evading the well behaved general bounds of [3, 4]) with important consequences for the Higgs phenomenology in such models.

The purpose of this letter is twofold. First of all, to re-analyze the SSMM bound. In section 1 we will show how the SSMM bound on the LHB tree-level mass can be improved, eliminating all the dangerous dependence on exotic VEVs [this will be true also after the inclusion of the most important one-loop radiative corrections (from top-stop and bottom-sbottom loops), see section 2]. And second of all, to re-derive in detail the general bound of Ref. [3] in order to clarify its applicability range. This will be done in section 3.

1. The Supersymmetric Singlet Majoron Model [6] is the simplest viable extension of the MSSM which can accommodate spontaneous R-parity breaking (and so, lepton number breaking). The extra fields added to the MSSM are right-handed neutrino chiral superfields N_i (with $L = -1$, and i is a family index, $i = 1, 2, 3$) and a singlet superfield Φ (with $L = 2$). With this field content, the most general superpotential renormalizable and gauge invariant is

$$f = \sum_{i,j} \left[h_{ij}^u Q_i \cdot H_2 U_j^c + h_{ij}^d H_1 \cdot Q_i D_j^c + h_{ij}^\nu L_i \cdot H_2 N_j + h_{ij}^e H_1 \cdot L_i E_j^c - \lambda_{ij} N_i N_j \Phi \right] + \mu H_1 \cdot H_2, \quad (1)$$

where the notation is self-explanatory. The new Yukawa couplings h_{ij}^ν are responsible for the mass of neutrinos and therefore they have to be small. For the phenomenological restrictions used to put bounds in these couplings see [6].

The tree-level scalar potential for neutral states, $V(H_1^0, H_2^0, \tilde{\nu}_i^0, \tilde{N}_j^0, \Phi^0)$ can be readily derived (where the $\tilde{\nu}_i^0$ are the left-handed sneutrinos). It consists of three parts:

$$V = V_F + V_D + V_{Soft}, \quad (2)$$

with

$$V_F = \left| \mu H_2^0 \right|^2 + \left| \mu H_1^0 + \sum_{i,j} h_{ij}^\nu \tilde{\nu}_i^0 \tilde{N}_j^0 \right|^2 + \sum_i \left| \sum_j h_{ij}^\nu H_2^0 \tilde{N}_j^0 \right|^2 + \sum_j \left| \sum_i (h_{ij}^\nu H_2^0 \tilde{\nu}_i^0 - 2\lambda_{ij} \tilde{N}_i^0 \Phi^0) \right|^2 + \left| \sum_{i,j} \lambda_{ij} \tilde{N}_i^0 \tilde{N}_j^0 \right|^2, \quad (3)$$

$$V_D = \frac{1}{8} G^2 \left[\left| H_1^0 \right|^2 - \left| H_2^0 \right|^2 + \sum_i \left| \tilde{\nu}_i^0 \right|^2 \right]^2, \quad (4)$$

where $G^2 \equiv g^2 + g'^2$, and

$$V_{Soft} = m_1^2 \left| H_1^0 \right|^2 + m_2^2 \left| H_2^0 \right|^2 + \sum_i m_{\tilde{N}_i}^2 \left| \tilde{N}_i^0 \right|^2 + m_\phi^2 \left| \Phi^0 \right|^2 + \sum_i m_{\tilde{\nu}_i}^2 \left| \tilde{\nu}_i^0 \right|^2 - \left[B\mu H_1^0 H_2^0 + \sum_{i,j} A_{ij}^\nu h_{ij}^\nu H_2^0 \tilde{\nu}_i^0 \tilde{N}_j^0 - \sum_{i,j} A_{ij}^\lambda \lambda_{ij} \tilde{N}_i^0 \tilde{N}_j^0 \Phi^0 + h.c. \right]. \quad (5)$$

For simplicity, we will neglect any possible CP breaking effects in the following assuming that all the parameters in this potential are real. Then we will take $\lambda_{ij} = \lambda_i \delta_{ij}$ by an appropriate rotation of the N_i fields. As was shown in ref. [6], in a wide region of parameter space, not only H_1^0 and H_2^0 develop a VEV but also $\tilde{\nu}_i^0$, \tilde{N}_j^0 and Φ^0 do:

$$\langle H_1^0 \rangle = v_1, \quad \langle H_2^0 \rangle = v_2, \quad \langle \tilde{\nu}_i^0 \rangle = x_i, \quad \langle \tilde{N}_j^0 \rangle = y_j, \quad \langle \Phi \rangle = \phi, \quad (6)$$

with the hierarchy $x_i \sim h^\nu v \ll v \equiv \sqrt{v_1^2 + v_2^2} \ll y_j, \phi \sim \mathcal{O}(M_S)$, (M_S representing the scale of soft supersymmetry breaking, *e.g.* $M_S \leq 1 \text{ TeV}$) thus leading to the spontaneous breaking

of $SU(2)_L \times U(1)_Y$, lepton number L and R-parity. The implications of this interesting scenario are further studied in [6, 7].

Following the general procedure, an upper bound on the tree level mass of the LHB in the SSMM can be obtained studying the scalar mass matrix for the fields $H_1^{0r}, H_2^{0r}, \tilde{\nu}_i^{0r}, \tilde{N}_j^{0r}, \Phi^{0r}$ [with $\phi_i^0 = (\phi_i^{0r} + i\phi_i^{0i})/\sqrt{2}$]. That matrix is now a 9×9 matrix but the analysis of the 2×2 submatrix for H_1^{0r}, H_2^{0r} leads easily to such a bound. This upper bound was calculated in [5] and is

$$m_h^2 \leq \frac{1}{2}G^2v^2 \cos^2 2\beta + \sum_i x_i^2 \left[\frac{m_{\tilde{\nu}_i}^2}{v^2} + \frac{1}{4}G^2 \left(\cos 2\beta + \sum_j \frac{x_j^2}{v^2} \right) \right] + \sum_{i,j,k,l} x_i x_j h_{ik}^\nu h_{jl}^\nu \left(\delta_{kl} \sin^2 \beta + \frac{y_k y_l}{v^2} \right), \quad (7)$$

with $v^2 \equiv v_1^2 + v_2^2$ and $\tan \beta \equiv v_2/v_1$. As stressed in [5], there is an explicit dependence of the bound on the exotic VEVs x_i and y_k and on the soft breaking mass of sneutrinos, $m_{\tilde{\nu}_i}$. In particular, the bound (7) is not finite in the formal limit $x_i, y_k, m_{\tilde{\nu}_i} \rightarrow \infty$, that is, there is no decoupling of the exotic VEVs and soft masses from the bound. The most important one loop corrections to this tree level bound, coming from the top-stop and bottom-sbottom sectors were also calculated in [5] and it was also found the same bad non-decoupling behaviour. Nevertheless, the smallness of h_{ij}^ν and x_i makes the bound numerically indistinguishable from the MSSM bound $M_Z^2 \cos^2 2\beta$.

Anyhow, as we are about to see, the bound (7) can in fact be improved, that is, (7) is not saturated and a more stringent bound can be found. Moreover, this new bound will turn out to be independent of soft-breaking masses or exotic VEVs and is always $\mathcal{O}(M_Z^2)$. In order to achieve this improvement we need to examine larger mass submatrices. In fact, we will need the 5×5 mass matrix for the fields H_1^{0r}, H_2^{0r} and $\tilde{\nu}_i^{0r}$. The elements of this matrix, derived from the effective potential (3-5) are:

$$\begin{aligned} M_{11}^2 &= \mu_1^2 + \frac{1}{4}G^2(3v_1^2 - v_2^2) + \frac{1}{4}G^2x^2 = -m_3^2 \tan \beta + \frac{1}{2}G^2v_1^2 - \mu \frac{\sigma^2}{v_1}, \\ M_{22}^2 &= \mu_2^2 - \frac{1}{4}G^2(v_1^2 - 3v_2^2) - \frac{1}{4}G^2x^2 + \sum_i \left(\sum_j h_{ij}^\nu y_j \right)^2 + \sum_j \left(\sum_i h_{ij}^\nu x_i \right)^2 \\ &= -m_3^2 \cot \beta + \frac{1}{2}G^2v_2^2 + \frac{R^3}{v_2}, \\ M_{12}^2 &= m_3^2 - \frac{1}{2}G^2v_1v_2, \end{aligned} \quad (8)$$

with $\mu_i^2 = |\mu|^2 + m_i^2$, $x^2 = \sum_i x_i^2$ and $m_3^2 = -B\mu$. The mass parameters μ_1 and μ_2 have been traded by the VEVs v_1 and v_2 using the minimization conditions $\partial V/\partial v_1 = \partial V/\partial v_2 = 0$:

$$\begin{aligned} \mu_1^2 &= -\frac{1}{4}G^2v^2 \cos 2\beta - m_3^2 \tan \beta - \mu \frac{\sigma^2}{v_1} - \frac{1}{4}G^2x^2, \\ \mu_2^2 &= \frac{1}{4}G^2v^2 \cos 2\beta - m_3^2 \cot \beta + \frac{R^3}{v_2} + \frac{1}{4}G^2x^2 - \sum_i \left(\sum_j h_{ij}^\nu y_j \right)^2 - \sum_j \left(\sum_i h_{ij}^\nu x_i \right)^2, \end{aligned} \quad (9)$$

where

$$\begin{aligned}\sigma^2 &\equiv \sum_{i,j} h_{ij}^\nu x_i y_j, \\ R^3 &\equiv \sum_{i,j} h_{ij}^\nu (2\lambda_j \phi + A_{ij}^\nu) x_i y_j.\end{aligned}\quad (10)$$

Due to the breaking of lepton number, Higgses and sneutrinos mix and the corresponding matrix elements are non vanishing:

$$\begin{aligned}M_{\tilde{\nu}_i 1}^2 &= \mu \sigma_i + \frac{1}{2} G^2 v_1 x_i, \\ M_{\tilde{\nu}_i 2}^2 &= -\sum_j h_{ij}^\nu y_j (2\lambda_j \phi + A_{ij}^\nu) - \frac{1}{2} G^2 v_2 x_i + 2v_2 \sum_{j,k} h_{ij}^\nu h_{kj}^\nu x_k,\end{aligned}\quad (11)$$

where $\sigma_i \equiv \sum_j h_{ij}^\nu y_j$. And finally we also need the sneutrino mass matrix

$$M_{\tilde{\nu}_i \tilde{\nu}_j}^2 = \left(m_{\tilde{\nu}_i}^2 + \frac{1}{4} G^2 x^2 + \frac{1}{4} G^2 v^2 \cos 2\beta \right) \delta_{ij} + \sigma_i \sigma_j + \frac{1}{2} G^2 x_i x_j + v_2^2 \sum_k h_{ik}^\nu h_{jk}^\nu. \quad (12)$$

For later use we also obtain from the condition $\partial V / \partial x_i = 0$ the relation

$$\sum_i m_{\tilde{\nu}_i}^2 x_i^2 = v_2 R^3 - \frac{1}{4} G^2 x^2 (x^2 + v^2 \cos 2\beta) - \mu v_1 \sigma^2 - \sigma^4 - v_2^2 \sum_j \left(\sum_i h_{ij}^\nu x_i \right)^2, \quad (13)$$

from which sneutrino masses can be traded by other parameters of the potential.

Next we define the normalized field $\tilde{\nu}^0$ as

$$\tilde{\nu}^0 \equiv \frac{\sum_i x_i \tilde{\nu}_i^0}{\sqrt{\sum_i x_i^2}} = \frac{1}{x} \sum_i x_i \tilde{\nu}_i^0, \quad (14)$$

such that $\langle \tilde{\nu}^0 \rangle = x$, while any combination of the fields $\tilde{\nu}_i^0$ orthogonal to $\tilde{\nu}^0$ has a vanishing VEV. We also define the new field $H_1'^0$ as the combination

$$H_1'^0 = \frac{v_1 H_1^0 + x \tilde{\nu}^0}{\sqrt{v_1^2 + x^2}} \equiv \frac{1}{v_1'} (v_1 H_1^0 + x \tilde{\nu}^0), \quad (15)$$

so that $\langle H_1'^0 \rangle = v_1' \equiv \sqrt{v_1^2 + x^2}$. This redefinition of fields amounts to a change of basis from $(H_1^{0r}, H_2^{0r}, \tilde{\nu}_i^{0r})$ to $(H_1'^{0r}, H_2'^{0r}, \dots)$ where now the only doublet fields having a non zero VEV are $H_1'^{0r}$ and $H_2'^{0r}$. The important point is that, in this rotated basis, the 2×2 mass submatrix (for $H_1'^{0r}, H_2'^{0r}$), $M_{ij}'^2$, has a simpler form. Using (13),

$$\begin{aligned}M_{11}'^2 &= \frac{1}{v_1'^2} \left[v_1^2 M_{11}^2 + 2v_1 \sum_i x_i M_{\tilde{\nu}_i 1}^2 + \sum_{i,j} x_i x_j M_{\tilde{\nu}_i \tilde{\nu}_j}^2 \right] \\ &= \frac{1}{v_1'^2} \left[-v_1^2 m_3^2 \tan \beta + \frac{1}{2} G^2 v_1'^4 + R^3 v_2 \right],\end{aligned}\quad (16)$$

$$M_{22}'^2 = M_{22}^2 = -m_3^2 \cot \beta + \frac{1}{2}G^2 v_2^2 + \frac{R^3}{v_2}, \quad (17)$$

$$\begin{aligned} M_{12}'^2 &= \frac{1}{v_1'} \left[v_1 M_{12}^2 + \sum_i x_i M_{\tilde{\nu}_i 2}^2 \right] \\ &= \frac{1}{v_1'} \left[m_3^2 v_1 - \frac{1}{2}G^2 v_1'^2 v_2 - R^3 + 2v_2 \sum_j \left(\sum_i h_{ij}^\nu x_i \right)^2 \right]. \end{aligned} \quad (18)$$

Or, writing

$$m_3'^2 \equiv \frac{1}{v_1'}(m_3^2 v_1 - R^3); \quad h_j'^2 \equiv \frac{1}{v_1'} \sum_i h_{ij}^\nu x_i; \quad \tan \beta' \equiv \frac{v_2}{v_1'}, \quad (19)$$

(note that this last definition is the natural one in this model) the 2×2 submatrix takes the form:

$$M^2 = \begin{vmatrix} -m_3'^2 \tan \beta' + \frac{1}{2}G^2 v_1'^2 & m_3'^2 - \frac{1}{2}G^2 v_1' v_2 + 2 \sum_i h_i'^2 v_1' v_2 \\ m_3'^2 - \frac{1}{2}G^2 v_1' v_2 + 2 \sum_i h_i'^2 v_1' v_2 & -m_3'^2 \cot \beta' + \frac{1}{2}G^2 v_2^2 \end{vmatrix}. \quad (20)$$

As is well known, the full 9×9 scalar mass matrix must have an eigenvalue smaller than (or equal to) the smallest eigenvalue of this 2×2 submatrix. In this way the following upper limit for the LHB mass results:

$$m_h^2 \leq \frac{1}{2}G^2 v'^2 \cos^2 2\beta' + v'^2 \sum_i h_i'^2 \sin^2 2\beta', \quad (21)$$

where $v'^2 \equiv v_1^2 + v_2^2 + x^2$. Note that now

$$M_Z^2 = \frac{1}{2}G^2 v'^2 = \frac{1}{2}(g^2 + g'^2) \left[v_1^2 + v_2^2 + \sum_i x_i^2 \right], \quad (22)$$

$[\langle \tilde{\nu}_i^0 \rangle = x_i$ breaks $SU(2)_L \times U(1)_Y$ and then contributes to the gauge boson masses] so that v' is fixed to be 174 GeV .

Remarkably, the bound (21) has the same form as the general bound calculated for supersymmetric models with an extended Higgs sector [3] and so it exhibits its same good properties, namely to be controlled exclusively by the electroweak scale (note that the rotated Yukawa couplings $h_j' = \sum_i h_{ij}^\nu x_i / v_1'$ are well behaved even in the formal limit $x_i \rightarrow \infty$ because $x_i / v_1' < 1$).

2. It is natural to ask whether the good effect of this field rotation also extends to radiative corrections. That is, do the one-loop radiative corrections to the bound (21) exhibit decoupling of the exotic VEVs x_i, y_k and soft-breaking masses $m_{\tilde{\nu}_i}$? We will show in this section that this is actually what happens for the most important radiative corrections: the ones coming from the top-stop and bottom-sbottom sectors.

The leading terms of the one-loop corrections to the Higgs mass can be easily calculated using the well known expression for the one-loop contribution to the effective potential (in \overline{DR} scheme)

$$\Delta V_1 = \frac{1}{64\pi^2} \text{Str} M_i^4 \left(\log \frac{M_i^2}{Q^2} - \frac{3}{2} \right), \quad (23)$$

where Q is the renormalization scale and M_i are the field-dependent masses of the different species of particles. To fix the notation we list here the relevant masses for the top-stop sector, which are given by

$$m_t^2(v_2) = h_t^2 v_2^2, \quad (24)$$

$$M_{\tilde{t}}^2(v_1, v_2, x_i, y_j) = \left\| \begin{array}{cc} m_{\tilde{Q}}^2 + m_t^2 + M_Z^2 D_L^t \cos 2\beta & h_t(A_t v_2 + \mu v_1 + \sigma^2) \\ h_t(A_t v_2 + \mu v_1 + \sigma^2) & m_{\tilde{U}}^2 + m_t^2 + M_Z^2 D_R^t \cos 2\beta \end{array} \right\|, \quad (25)$$

with $m_{\tilde{Q}}^2, m_{\tilde{U}}^2$ the soft masses for left and right-handed stops, A_t the trilinear coupling associated with the term $h_t Q_3 \cdot H_2 U_3^c$ in the superpotential and

$$D_L^t = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, \quad D_R^t = \frac{2}{3} \sin^2 \theta_W, \quad (26)$$

while σ^2 was defined in (10). We will call $m_{t_1}^2, m_{t_2}^2$ the two eigenvalues of (25) with $m_{t_1}^2 \geq m_{t_2}^2$.

For the bottom-sbottom sector we have

$$m_b^2(v_1) = h_b^2 v_1^2, \quad (27)$$

$$M_{\tilde{b}}^2(v_1, v_2) = \left\| \begin{array}{cc} m_{\tilde{Q}}^2 + m_b^2 + M_Z^2 D_L^b \cos 2\beta & h_b(A_b v_1 + \mu v_2) \\ h_b(A_b v_1 + \mu v_2) & m_{\tilde{D}}^2 + m_b^2 + M_Z^2 D_R^b \cos 2\beta \end{array} \right\|, \quad (28)$$

with $m_{\tilde{D}}^2$ the soft mass for right-handed sbottoms, A_b the trilinear coupling associated with the term $h_b H_1 \cdot Q_3 D_3^c$ in the superpotential and

$$D_L^b = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad D_R^b = -\frac{1}{3} \sin^2 \theta_W. \quad (29)$$

We will call $m_{b_1}^2, m_{b_2}^2$ the two eigenvalues of (28) with $m_{b_1}^2 \geq m_{b_2}^2$. As we are not considering the gauge contributions to (23), we will also neglect the D term contributions to (25) and (28) in the following.

The one-loop corrected effective potential is a function of $\phi_i = (H_1^{0r}, H_2^{0r}, \tilde{\nu}_i^{0r}, \tilde{N}_j^{0r}, \phi^{0r})$ [or equivalently $(v_1, v_2, x_i, y_j, \phi)$] and so, the squared-mass matrix M_{ij}^2 will receive a one-loop correction $\partial^2 \Delta V_1 / \partial \phi_i \partial \phi_j$. After correcting also the minimization conditions (9) by including the one-loop contribution $\partial \Delta V_1 / \partial \phi_i$ we obtain

$$\begin{aligned} M_{11}^{2(1)} &= -(m_3^2 + \delta m_3^2) \tan \beta + \frac{1}{2} G^2 v_1^2 - \frac{1}{v_1} \mu \sigma^2 (1 + \delta f) + \Delta_{11}, \\ M_{22}^{2(1)} &= -(m_3^2 + \delta m_3^2) \cot \beta + \frac{1}{2} G^2 v_2^2 + \frac{R^3}{v_2} - \frac{A_t}{v_2} \sigma^2 \delta f + \Delta_{22}, \\ M_{12}^{2(1)} &= (m_3^2 + \delta m_3^2) - \frac{1}{2} G^2 v_1 v_2 + \Delta_{12}, \end{aligned} \quad (30)$$

with

$$\begin{aligned}
\delta m_3^2 &= \frac{3}{32\pi^2} \left\{ \frac{h_t^2 A_t \mu}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} [f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)] + (t \rightarrow b) \right\}, \\
\delta f &= \frac{3h_t^2}{32\pi^2} \frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}, \\
\Delta_{11} &= \frac{3}{8\pi^2} \left\{ h_t^2 m_t^2 \mu^2 A_T'^2 g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right. \\
&\quad \left. + h_b^2 m_b^2 \left[\log \frac{m_{b_1}^2 m_{b_2}^2}{m_b^4} + 2A_b A_B \log \frac{m_{b_1}^2}{m_{b_2}^2} + A_b^2 A_B^2 g(m_{b_1}^2, m_{b_2}^2) \right] \right\}, \\
\Delta_{22} &= \frac{3}{8\pi^2} \left\{ h_b^2 m_b^2 \mu^2 A_B^2 g(m_{b_1}^2, m_{b_2}^2) \right. \\
&\quad \left. + h_t^2 m_t^2 \left[\log \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} + 2A_t A_T' \log \frac{m_{t_1}^2}{m_{t_2}^2} + A_t^2 A_T'^2 g(m_{t_1}^2, m_{t_2}^2) \right] \right\}, \\
\Delta_{12} &= \frac{3}{8\pi^2} \left\{ h_t^2 m_t^2 \mu A_T' \left[\log \frac{m_{t_1}^2}{m_{t_2}^2} + A_t A_T' g(m_{t_1}^2, m_{t_2}^2) \right] \right. \\
&\quad \left. + h_b^2 m_b^2 \mu A_B \left[\log \frac{m_{b_1}^2}{m_{b_2}^2} + A_b A_B g(m_{b_1}^2, m_{b_2}^2) \right] \right\},
\end{aligned} \tag{31}$$

and

$$f(m^2) = 2m^2 \left[\log \frac{m^2}{Q^2} - 1 \right], \quad g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}, \tag{32}$$

$$A_T' = \frac{1}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(A_t + \mu \cot \beta + \frac{\sigma^2}{v_2} \right), \quad A_B = \frac{A_b + \mu \tan \beta}{m_{b_1}^2 - m_{b_2}^2}. \tag{33}$$

All of the matrix elements $M_{ij}^{2(1)}$ ($i, j = 1, 2$) diverge in the decoupling SUSY limit $\mu y_k, m_3^2 + \delta m_3^2 \rightarrow \infty$. In fact, writing $\mu^2 \sim y_k^2 \sim m_3^2 + \delta m_3^2 \sim \mathcal{O}(M_S^2)$, all the $M_{ij}^{2(1)}$ grow like M_S^2 :

$$\begin{aligned}
M_{11}^{2(1)} &= -(m_3^2 + \delta m_3^2) \tan \beta - \frac{1}{v_1} \mu \sigma^2 (1 + \delta f) + \dots, \\
M_{22}^{2(1)} &= -(m_3^2 + \delta m_3^2) \cot \beta + \frac{R^3}{v_2} - \frac{A_t}{v_2} \sigma^2 \delta f + \dots, \\
M_{12}^{2(1)} &= (m_3^2 + \delta m_3^2) + \dots,
\end{aligned} \tag{34}$$

where the dots stand for contributions that are finite in the decoupling limit. The eigenvalues of this matrix will grow also like M_S^2 if $\text{Det} M^{2(1)}$ grows like M_S^4 , but, if due to some

cancellation, $DetM^{2(1)}$ grows only like M_S^2 one of the eigenvalues (the lightest) will remain finite in the decoupling limit [8]. In fact one can see that this cancellation takes place for the $(m_3^2 + \delta m_3^2)^2$ contribution to $DetM^{2(1)}$ but not for the rest of the terms. Then, as was found in [5], the bound derived from this unrotated submatrix receives one-loop corrections which are not controlled only by the electroweak scale.

To study the one-loop rotated matrix we need also

$$\begin{aligned} M_{\tilde{\nu}_i 1}^2{}^{(1)} &= M_{\tilde{\nu}_i 1}^2 + \mu \sigma_i \eta, \\ M_{\tilde{\nu}_i 2}^2{}^{(1)} &= M_{\tilde{\nu}_i 2}^2 + A_t \sigma_i \eta + \frac{3}{8\pi^2} h_t^2 m_t^2 A'_T \sigma_i \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}, \\ M_{\tilde{\nu}_i \tilde{\nu}_j}^2{}^{(1)} &= M_{\tilde{\nu}_i \tilde{\nu}_j}^2 + \sigma_i \sigma_j \eta, \end{aligned} \quad (35)$$

with the tree level matrix elements on the right hand side as given in (11) and (12) and

$$\eta = \frac{3h_t^2}{8\pi^2} m_t^2 A'^2 g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \delta f. \quad (36)$$

Note that $\sum_{i,j} M_{\tilde{\nu}_i \tilde{\nu}_j}^2 x_i x_j$ contains the term $\sum_i m_{\tilde{\nu}_i}^2 x_i^2$ which receives also the one-loop correction

$$\delta \sum_i m_{\tilde{\nu}_i}^2 x_i^2 = -\frac{1}{2} \sum_i x_i \frac{\partial \Delta V_1}{\partial x_i} = -\frac{3h_t^2}{32\pi^2} v_2 A'_T \sigma^2 [f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)]. \quad (37)$$

The rotated 2×2 mass matrix then takes the one-loop form:

$$M'^2 = \begin{vmatrix} -(m_3'^2 + \delta m_3'^2) \tan \beta' + \Delta'_{11} & (m_3'^2 + \delta m_3'^2) + \Delta'_{12} \\ (m_3'^2 + \delta m_3'^2) + \Delta'_{12} & -(m_3'^2 + \delta m_3'^2) \cot \beta' + \Delta'_{22} \end{vmatrix}. \quad (38)$$

with

$$\delta m_3'^2 = \frac{1}{v_1'} [\delta m_3^2 v_1 + A_t \sigma^2 \delta f]. \quad (39)$$

The terms Δ'_{ij} are finite in the decoupling limit and are given by:

$$\begin{aligned} \Delta'_{11} &= \frac{1}{2} G^2 v_1'^2 + \frac{3}{8\pi^2} h_t^2 m_t^2 A'^2 \frac{\sigma^2}{v_1'^2} (\sigma^2 + 2\mu v_1) g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{v_1^2}{v_1'^2} \Delta_{11}, \\ \Delta'_{22} &= \frac{1}{2} G^2 v_2^2 + \Delta_{22}, \\ \Delta'_{12} &= -\frac{1}{2} G^2 v_1' v_2 + 2 \sum_j h_j'^2 v_1' v_2 + \frac{3}{8\pi^2} \frac{1}{v_1'} h_t^2 m_t^2 \sigma^2 \eta A_t A'^2 g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \\ &\quad + \frac{3}{8\pi^2} \frac{1}{v_1'} h_t^2 m_t^2 \sigma^2 A'_T \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{v_1}{v_1'} \Delta_{12}. \end{aligned} \quad (40)$$

A look at (38) shows immediately that there is decoupling, that is, $DetM'^2 = M_{11}'^2 M_{22}'^2 - M_{12}'^4 \sim \mathcal{O}(M_S^2)$.

In fact, the one-loop corrected version of the tree-level bound (21) takes the simple form

$$m_h^2 \leq \Delta'_{11} \cos^2 \beta' + \Delta'_{22} \sin^2 \beta' + \Delta'_{12} \sin 2\beta' = M_Z^2 \cos^2 2\beta' + 2M_W^2 \sum_i \frac{h_i'^2}{g^2} \sin^2 2\beta' \\ + \frac{3g^2}{16\pi^2} \left\{ \frac{m_t^4}{m_W^2} \left[\log \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} + \tilde{X}_t^2 \left(2 \log \frac{m_{t_1}^2}{m_{t_2}^2} + \tilde{X}_t^2 g(m_{t_1}^2, m_{t_2}^2) \right) \right] + (t \rightarrow b) \right\}, \quad (41)$$

where

$$\tilde{X}_t^2 = \frac{(A_t + \mu \cot \beta + \sigma^2/v_2)^2}{m_{t_1}^2 - m_{t_2}^2}; \quad \tilde{X}_b^2 = \frac{(A_b + \mu \tan \beta)^2}{m_{b_1}^2 - m_{b_2}^2}. \quad (42)$$

Note that the one-loop corrections have exactly the same form as in the MSSM, the only difference arising in \tilde{X}_t^2 which now includes the term σ^2/v_2 .

3. In this section we will re-derive the bound on the lightest Higgs boson mass for general Supersymmetric Standard Models under very general assumptions. The particular form of the bound for some special cases of interest [3] will be presented, and at the end we will show how to apply the general bound to the SSMM case finding the same result obtained in the direct calculation of section 1.

Let us first assume that the Higgs sector of the general Supersymmetric model under consideration contains at least two $SU(2)_L$ doublets (H_1, H_2 with hypercharges $\pm 1/2$) taking vacuum expectation values

$$\left. \begin{aligned} \langle H_1 \rangle &= v_1 \\ \langle H_2 \rangle &= v_2 \end{aligned} \right\} \quad v^2 \equiv v_1^2 + v_2^2 \leq (174 \text{ GeV})^2, \quad (43)$$

with the equality holding when only these two doublets drive electroweak breaking. In the general case other fields may contribute to gauge boson masses. When there are extra doublets (d doublets with hypercharge $-1/2$ and d with hypercharge $1/2$) it is well known that a field rotation can be made such that only one doublet of each type takes a non zero VEV [9] (that can be taken real and positive if electric charge is conserved). In that case these two rotated fields will be the ones we are calling H_1 and H_2 (as we did for the SSMM in the previous sections). In general other fields in higher $SU(2)_L$ representations can participate in the electroweak breaking (but satisfying the constraints from $\Delta\rho$). In the following we will make the reasonable assumption that no fields in representations higher than triplets take non-zero VEVs¹ (models with unsuppressed triplet VEVs have been studied in the literature).

In general the LHB has the same quantum numbers as the Standard Model Higgs so that we will concentrate in the study of the CP even Higgs sector². Let us consider the most

¹The vast majority of the models of interest satisfy this requirement although a bound that applies even without this restriction can be derived (see Ref. [10]).

²This terminology is only valid when CP is a good symmetry (as in the MSSM), but we do not need this assumption for the derivation to be correct.

general tree-level potential for the fields H_1^{0r}, H_2^{0r} (coming from a renormalizable full scalar potential):

$$\begin{aligned}
V = & V_0 + M_1^3 H_1^{0r} + M_2^3 H_2^{0r} \\
& + m_1^2 (H_1^{0r})^2 + 2m_{12}^2 H_1^{0r} H_2^{0r} + m_2^2 (H_2^{0r})^2 \\
& + M_{11} (H_1^{0r})^3 + M_{12} (H_1^{0r})^2 H_2^{0r} + M_{21} (H_2^{0r})^2 H_1^{0r} + M_{22} (H_2^{0r})^3 \\
& + \lambda_1 (H_1^{0r})^4 + \lambda_{12} (H_1^{0r})^2 (H_2^{0r})^2 + \lambda_2 (H_2^{0r})^4 \\
& + \lambda'_{12} (H_1^{0r})^3 H_2^{0r} + \lambda'_{21} H_1^{0r} (H_2^{0r})^3,
\end{aligned} \tag{44}$$

In general, the parameters appearing in this potential are some function of other scalar fields, *e.g.* $M_1 \equiv M_1(\chi^{0r}, \xi^{0r}, \dots)$. The superindex o will indicate that such functions have to be evaluated with these extra scalar fields at their vacuum expectation values: $M_1^o \equiv M_1(\chi^{0r} = \langle \chi^{0r} \rangle, \xi^{0r} = \langle \xi^{0r} \rangle, \dots)$.

By symmetry considerations some of the terms in this potential can be forbidden. For example, imposing invariance of the potential under the symmetry transformation $H_i^{0r} \rightarrow -H_i^{0r}$ many couplings in (44) should be set to zero. As we will see later, we do not need to impose this symmetry by hand but it will arise automatically for the quartic couplings of the supersymmetric theories in which we are interested.

As H_1^{0r}, H_2^{0r} are the neutral components (more precisely the real part) of two $SU(2)_L$ doublets and the full potential has to be gauge invariant, it can be immediately deduced that $M_{1,2}^3$ must come from a field or combination of fields transforming as a $SU(2)_L$ doublet, that is

$$(M_{1,2}^o)^3 \sim m^2 \langle d \rangle, \tag{45}$$

with m^2 some gauge invariant squared mass and $\langle d \rangle$ representing the VEV of some $SU(2)_L$ doublet (fundamental or not, but different from H_1^{0r} and H_2^{0r} by construction). Now, as H_1 and H_2 are the only doublets with a non vanishing VEV (and no fields in representations higher than triplets take a non-zero VEV) we get the restriction

$$(M_1^o)^3 = (M_2^o)^3 = 0. \tag{46}$$

Using similar arguments, $SU(2)_L \times U(1)_Y$ invariance of the full potential implies³

$$M_{ij}^o \sim \langle d \rangle, \tag{47}$$

so that one can also deduce the relations

$$M_{11}^o = M_{12}^o = M_{21}^o = M_{22}^o = 0. \tag{48}$$

Next, taking into account that the scalar potentials we are considering come from a (softly broken) supersymmetric theory, we will get some restrictions on the quartic couplings in V . As is well known, these quartic couplings are of fundamental importance for the tree-level upper bound we want to derive. Let us consider separately the two different types of supersymmetric contributions to the effective potential.

³By assumption we discard the possibility of M_{ij}^o transforming as a $SU(2)$ quadruplet and taking a VEV.

i) F terms

These are given by the well known formula

$$V_F = \sum_i \left| \frac{\partial f}{\partial \phi_i} \right|^2, \quad (49)$$

with $f(\phi_i) = W(\hat{\phi}_i \rightarrow \phi_i)$ [$W(\hat{\phi}_i)$ is the superpotential, $\hat{\phi}_i$ the chiral superfields and ϕ_i their scalar components]. The superpotential W is at most cubic in the superfields implying that the quartic F -terms in the potential must come from the cubic terms in W . It is easy to see that quartic terms like $(H_1^{0r})^3 H_2^{0r}$ or $(H_2^{0r})^3 H_1^{0r}$ can only arise if both a superfield $\hat{\phi}_i$ and its hermitian conjugate $\hat{\phi}_i^*$ appear in W , which is not possible (W being an analytic function of the chiral superfields). This proves that F -terms do not contribute to the λ'_{ij} couplings.

ii) D terms

The contribution of D terms to the scalar potential takes the form

$$V_D = \frac{1}{2} \sum_a g_a^2 \left| \phi_i T_{ij}^a \phi_j^* \right|^2, \quad (50)$$

where a runs over the gauge groups (with coupling constant g_a) and T^a are the generating matrices of group a in the representation of the fields ϕ_i . According to (50) the only quartic terms obtained are of the form $H_1 H_1 H_1^* H_1^*$, $H_1 H_2 H_1^* H_2^*$ or $H_2 H_2 H_2^* H_2^*$. So, neither D terms nor F terms contribute to the λ' couplings and then

$$\lambda'_{12} = \lambda'_{21} = 0. \quad (51)$$

Inserting eqs. (46), (48) and (51) in (44), the tree-level scalar potential for H_1^{0r} , H_2^{0r} takes the following form

$$\begin{aligned} V_S(H_1^{0r}, H_2^{0r}) &= V_0^o + \frac{1}{2}(m_1^o)^2 (H_1^{0r})^2 + (m_{12}^o)^2 H_1^{0r} H_2^{0r} + \frac{1}{2}(m_2^o)^2 (H_2^{0r})^2 \\ &+ \frac{1}{4}\lambda_{11}(H_1^{0r})^4 + \frac{1}{4}\lambda_{12}(H_1^{0r})^2 (H_2^{0r})^2 + \frac{1}{4}\lambda_{22}(H_2^{0r})^4. \end{aligned} \quad (52)$$

After writing $H_i^{0r} = h_i^{0r} + \sqrt{2}v_i$ [which corresponds to $\langle H_i^0 \rangle \equiv \langle (H_i^{0r} + iH_i^{0i})/2 \rangle = v_i$], m_1^o and m_2^o can be expressed as functions of the other parameters of the potential just imposing that V_S has its minimum at (v_1, v_2) , that is, using the minimization conditions

$$\frac{\partial V_S}{\partial h_i^{0r}} = 0, \quad (i = 1, 2). \quad (53)$$

In that way we are led to

$$\begin{aligned} m_1^2 &= -m_{12}^2 \tan \beta - 2\lambda_{11}v_1^2 - \lambda_{12}v_2^2, \\ m_2^2 &= -m_{12}^2 \cot \beta - 2\lambda_{22}v_2^2 - \lambda_{12}v_1^2, \end{aligned} \quad (54)$$

where $\tan \beta = v_2/v_1$ and the superindex o is omitted for simplicity.

The analysis of the mass submatrix (\mathcal{M}^2) for the fields h_1^{0r}, h_2^{0r} will give us the mass bound on the LHB mass. This submatrix is now:

$$\begin{aligned}\mathcal{M}^2 &= \begin{vmatrix} m_1^2 + 6\lambda_{11}v_1^2 + \lambda_{12}v_2^2 & m_{12}^2 + 2\lambda_{12}v_1v_2 \\ m_{12}^2 + 2\lambda_{12}v_1v_2 & m_2^2 + 6\lambda_{22}v_2^2 + \lambda_{12}v_1^2 \end{vmatrix} \\ &= \begin{vmatrix} -m_{12}^2 \tan \beta + 4\lambda_{11}v_1^2 & m_{12}^2 + 2\lambda_{12}v_1v_2 \\ m_{12}^2 + 2\lambda_{12}v_1v_2 & -m_{12}^2 \cot \beta + 4\lambda_{22}v_2^2 \end{vmatrix},\end{aligned}\quad (55)$$

where the minimization conditions (54) have been used to write the second expression. It is simple to show that the eigenvalues of this matrix can be written as

$$\begin{aligned}m_{\pm}^2 &= \frac{1}{2} \left\{ -\bar{m}_{12}^2 + 4\lambda_{11}v_1^2 + 4\lambda_{22}v_2^2 \pm \left[\left(\bar{m}_{12}^2 + 4v_{11}^2 - 4v_{22}^2 \right)^2 \cos^2 2\beta \right. \right. \\ &\quad \left. \left. + \left(\bar{m}_{12}^2 + 2\lambda_{12}v^2 \right)^2 \sin^2 2\beta \right]^{1/2} \right\},\end{aligned}\quad (56)$$

with $\bar{m}_{12}^2 \equiv 2m_{12}^2 / \sin 2\beta$ and $v_{ii}^2 \equiv \lambda_{ii}v_i^2 / \cos 2\beta$. Using the inequality

$$\left[a^2 \cos^2 2\beta + b^2 \sin^2 2\beta \right]^{1/2} \geq a \cos^2 2\beta + b \sin^2 2\beta, \quad (57)$$

the following bound results:

$$m_h^2 \leq m_-^2 \leq \left(4\lambda_{11} \cos^4 \beta + 4\lambda_{22} \sin^4 \beta + \lambda_{12} \sin^2 2\beta \right) v^2, \quad (58)$$

which is the central formula we were looking for.

Note that the bound is determined by the quartic couplings, as anticipated. It implies that the bound is not sensitive to the details of the soft-breaking. Then the only scale that enters the bound is v , the electroweak scale and so, even if the soft breaking scale gets large there is always a light scalar Higgs [that is, of mass $\mathcal{O}(M_Z)$] in the spectrum. One can go beyond (58) and obtain the particular form of the bound in some classes of models:

3.1 Models with an extended Higgs sector.

Having obtained the general formula (58), we can now find the particular form of the Higgs mass bound in a class of nonminimal Supersymmetric models in which the Higgs sector is extended and contains, apart from the two MSSM Higgs doublets, H_1, H_2 , other extra fields (singlets, more doublets without VEVs, triplets, etc. See refs. [11-15]).

In that class of models the λ_{ij} couplings in (58) come from:

i) F terms

The only cubic terms in the superpotential that can give a contribution are of the form

$$\Delta W \sim \lambda \hat{\phi} \hat{H}_i \hat{H}_j, \quad (59)$$

with $i, j = 1, 2$, and $\hat{\phi}$ an extra chiral superfield (note that there are no such terms in the MSSM). More specifically with a superpotential containing a part like

$$f = \lambda_0 \phi_0 H_1^0 H_2^0 + \frac{1}{2} \lambda_1 \phi_1 H_1^0 H_1^0 + \frac{1}{2} \lambda_{-1} \phi_{-1} H_2^0 H_2^0 + \dots, \quad (60)$$

the λ_{ij} couplings in (52) receive the contributions

$$\delta\lambda_{11} = \frac{1}{4}\lambda_1^2, \quad \delta\lambda_{22} = \frac{1}{4}\lambda_{-1}^2, \quad \delta\lambda_{12} = \lambda_0^2. \quad (61)$$

On top of that, note that $SU(2)_L \times U(1)_Y$ invariance of the superpotential requires the ϕ_k scalars to be singlets or neutral components of triplets with hypercharge $Y = 0, \pm 1$.

ii) D terms

The contributions are exactly the same as in the MSSM:

$$\begin{aligned} \delta\lambda_{11} &= \delta\lambda_{22} = \frac{1}{8}(g^2 + g'^2), \\ \delta\lambda_{12} &= -\frac{1}{4}(g^2 + g'^2), \end{aligned} \quad (62)$$

so that the bound on the LHB in these non-minimal models is

$$\frac{m_h^2}{v^2} \leq \frac{1}{2}(g^2 + g'^2) \cos^2 2\beta + \lambda_0^2 \sin^2 2\beta + \lambda_1^2 \cos^4 \beta + \lambda_{-1}^2 \sin^4 \beta, \quad (63)$$

with $v^2 \equiv v_1^2 + v_2^2$ as usual. This bound was first obtained in [3].

Of course, if $\lambda_k = 0$ the MSSM bound is recovered. The non-minimal correction, dependent on the new Yukawa couplings, λ_k , is positive definite so that the bound is weaker than in the MSSM. Also note that it is necessary to write the bound with an explicit v^2 dependence instead of using the formula $(g^2 + g'^2)v^2/2 = M_Z^2$. This relation between $v_1^2 + v_2^2$ and M_Z^2 will no longer be valid in models with an extended Higgs sector if other fields apart from the two doublets H_1 and H_2 contribute with their VEVs to M_Z^2 (*e.g.* triplets). If this happens, then $(g^2 + g'^2)v^2/2 < M_Z^2$ and the bound will be more restrictive.

3.2 Models with an extended gauge sector.

In a similar manner, an upper bound on the LHB mass can be obtained in Supersymmetric models which gauge group (at low energy, that is $\sim 1 \text{ TeV}$) is different from the Standard one [3, 8, 16, 17]. Usually, this type of models require the introduction of extra representations in the Higgs sector to give a correct gauge symmetry breaking and of extra fermions to cancel anomalies. The influence of the extra Higgs representations on the lightest Higgs bound has been considered in the previous subsection while the presence of extra exotic fermions can affect the bound through radiative corrections [18]. In particular models all these effects have to be combined. In this subsection we concentrate in the modifications of the tree level bound when the two doublets H_1, H_2 transform non trivially under the extra gauge groups.

Examining the D terms the following contributions to the λ_{ij} couplings in (52) are obtained:

$$\begin{aligned} \delta\lambda_{11} &= \frac{1}{2} \sum_a g_a^2 (T_1^a)^2, \quad \delta\lambda_{22} = \frac{1}{2} \sum_a g_a^2 (T_2^a)^2, \\ \delta\lambda_{12} &= - \sum_a g_a^2 T_1^a T_2^a, \end{aligned} \quad (64)$$

with

$$T_i^a \equiv \frac{\langle H_i T^a H_i^* \rangle}{\langle H_i H_i \rangle}, \quad (i = 1, 2). \quad (65)$$

This translates in the following modification of the bound on m_h^2 [3]:

$$\Delta m_h^2 = 2v^2 \sum_a g_a^2 \left[T_1^a \cos^2 \beta - T_2^a \sin^2 \beta \right]^2. \quad (66)$$

As a check, for $SU(2)_L$

$$T_1^L = T_2^L = \frac{1}{2}, \quad (67)$$

gives $\Delta m_h^2 = (g^2 v^2 / 2) \cos^2 2\beta$. And for $U(1)_Y$

$$T_1^Y = T_2^Y = \frac{1}{2}, \quad (68)$$

so that $\Delta m_h^2 = (g'^2 v^2 / 2) \cos^2 2\beta$, in agreement with the MSSM bound.

After the general discussion in this section it should be clear that one can apply the bound (63) also to the SSMM. Making use of the field rotation (15) directly in the superpotential (1) we get the term

$$h_{ij}^\nu L_i \cdot H_2 N_j = h_{ij}^\nu \frac{x_i}{v_1'} N_j H_1'^0 H_2^0 N_j^0 + \dots, \quad (69)$$

i.e. a λ_0 -type coupling which, according to (60) and (63) contributes to the bound with

$$\Delta m_h^2 = v'^2 \sum_j \left(\sum_i h_{ij}^\nu \frac{x_i}{v_1'} \right)^2 \sin^2 2\beta'. \quad (70)$$

This is the same result that was obtained by the direct calculation. Concerning the D term contribution to the bound, note that the effect of the field rotation (15) on V_D is

$$\begin{aligned} V_D &= \frac{1}{8} G^2 \left[|H_1^0|^2 - |H_2^0|^2 + \sum_i |\tilde{\nu}_i^0|^2 \right]^2 \\ &= \frac{1}{8} G^2 \left[\frac{v_1^2}{v_1'^2} |H_1^0|^2 - |H_2^0|^2 + \sum_i \frac{x_i^2}{v_1'^2} |\tilde{\nu}_i^0|^2 \right]^2 + \dots \\ &= \frac{1}{8} G^2 \left[|H_1'^0|^2 - |H_2^0|^2 \right]^2 + \dots, \end{aligned} \quad (71)$$

which is formally equivalent to the MSSM result with the replacement $H_1^0 \rightarrow H_1'^0$ and so gives the contribution

$$\Delta m_h^2 = \frac{1}{2} G^2 (v_1'^2 + v_2^2) \cos^2 2\beta', \quad (72)$$

in agreement with (21).

4. In conclusion, we have improved the tree-level upper bound on the mass of the lightest Higgs boson in the Supersymmetric Singlet Majoron Model, finding a new bound which is

controlled by the electroweak scale and remains light in the limit of heavy exotic VEVs or soft breaking masses that decouple from the bound. We have also proved (computing the most important one-loop corrections to this bound) that this decoupling is not spoiled by radiative corrections.

The similarity of the improved bound calculated in this paper for the lightest Higgs boson mass in the Supersymmetric Singlet Majoron Model with previous bounds derived for general Supersymmetric models with an extended Higgs sector has motivated the re-analysis of the derivation of these general bounds in order to clarify its range of applicability. We have shown that those general bounds are in fact based on very general assumptions, namely that the model contains a pair of doublets participating in the electroweak breaking and no fields in $SU(2)_L$ representations higher than triplets take a VEV. With this simple starting input and using gauge symmetry and supersymmetry to constrain the effective potential one is able to obtain a bound on the mass of the lightest Higgs boson of the theory. As a particular example we have re-derived the bound in the Supersymmetric Singlet Majoron Model using these general results.

It would be interesting to study whether the decoupling of exotic scales in one-loop radiative corrections to this tree-level bound is general, that is, to see if it is automatic for the class of models in which the tree level bound applies or if further assumptions are required. This subject is currently under investigation [19].

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